HONORS PRE-CALCULUS

CURRICULUM

CARLISLE AREA SCHOOL DISTRICT

DATE OF BOARD APPROVAL: **February 16, 2023**

COURSE OVERVIEW

Title:	Honors Pre-Calculus
Grade Level:	Grades 9-12
Level:	High School – Honors
Length:	Full Year
Duration:	85 Minute Periods
Frequency:	90 Days
Pre-Requisites:	Algebra 2 and Geometry
Credit:	1 Credit
Description:	Pre-Calculus is a course that extends and builds upon topics covered in Algebra 2 and introduces students to trigonometric concepts. By the end of this course, students will be able to solve and graph polynomial, exponential, logarithmic, and trigonometric functions. Students will also understand the unit circle, be able to apply trigonometric identities, and solve real world problems using trigonometric relationships.

COURSE TIMELINE

UNIT	TITLE	KEY CONCEPTS	DURATION (DAYS)
1	Functions and Graphs	• Domain and range • Parent graphs and transformations of functions • Analyzing graphs of functions • Combinations of functions • Inverses of functions	8 Days
2	Polynomial and Rational Functions	• Quadratic functions • Polynomial functions of higher degree (end behavior and intervals of increasing/decreasing) • Polynomial long and synthetic division • Zeros of polynomial functions • Rational functions • Nonlinear inequalities • Partial fractions	9 Days
3	Matrices	• Operations with matrices • Matrices and systems of equations • Inverse of a square matrix • Determinant of a square matrix • Applications of matrices and determinants	7 Days
4	Exponential and Logarithmic Functions	• Exponential functions • Logarithmic functions • Properties of logarithms • Exponential and logarithmic equations • Exponential and logarithmic models	8 Days
5	Introduction to Trigonometry	• Right triangle trigonometry • Radian and degree measure • Trigonometric functions and the unit circle • Trigonometric functions of any angle	9 Days

6	Trigonometric Graphs	• Trigonometric parent graphs • Trigonometric graph transformations • Inverse trigonometric functions • Trigonometric graph applications and models	9 Days
7	Trigonometric Identities	• Fundamental trigonometric identities • Verifying trigonometric identities • Solving trigonometric equations • Sum and difference formulas • Advanced trigonometric identities	10 Days
8	Polar Graphs	• Polar coordinates • Polar and rectangular conversion • Graphs of polar equations	7 Days
9	Sequences, Series, and Probability	• Sequences and series • Arithmetic sequences and partial sums • Geometric sequences and series • The Binomial Theorem • Combinations and permutations • Probability	8 Days
10	Introduction to Calculus	• Basic properties of limits • Direct substitution • Techniques of evaluating limits • One-sided limits	4 Days

DISCIPLINARY SKILLS and PRACTICES

DISCIPLINARY SKILL/PRACTICE	DESCRIPTION
Make sense of problems and persevere in solving them.	Make conjectures about how application problems may be solved, monitor progress toward a solution, and make adjustments in the problem-solving plan if necessary.
Reason abstractly and quantitatively.	Check solutions found to problems against educated guesses/estimates to that problem, in order to determine the reasonableness of the solution found.
Construct viable arguments and critique the reasoning of others.	Justify and communicate conclusions effectively and respond to arguments logically.
Model with mathematics.	Use mathematics to model real world problems, interpreting the mathematical results in the context of the situation.
Use appropriate tools strategically.	Consider the tools available in solving problems and understand the benefits and insights gained by using the tool as well as the limitation of the tool.
Attend to precision.	Calculate accurately and efficiently within the context of problems and communicate results precisely.
Look for and make use of structure.	Examine problems to discern a pattern or structure and utilize this finding to solve similar problems.
Look for and express regularity in repeated reasoning.	Notice repeated calculations or processes and generalize from those insights in order to solve problems.

UNIT 1

Unit Title	Functions and Graphs
Unit Description	Students will learn that functions show how one variable is related to another variable. Students will use graphical and algebraic methods to combine functions including how transformations can be applied to a library of parent functions. Functions are used to estimate values, simulate processes, and discover relationships.
Unit Assessment	Unit assessment

Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How do we find domain and range of relations and functions graphically and algebraically? **1 Day**	☐ Recognize and use interval notation for domain and range. ☐ Differentiate between bounded intervals and unbounded intervals. ☐ Find the domain and range from an equation.	**Vocabulary:** domain, range, function, interval notation	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
How do you apply the rules of transformations to write and graph new functions from known parent functions? **2 Days**	☐ Distinguish the different types of parent graphs by shape. ☐ Sketch graphs using transformation rules. ☐ Write equations using transformation form.	**Vocabulary:** linear, quadratic, cubic, absolute value, square root, cubed root, rational, step functions, vertical/horizontal shifts (rigid), reflection (rigid), vertical and horizontal stretch/compressions (non-rigid)	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.

How is the behavior of a function analyzed both algebraically and graphically? **2 Days**	☐ Classify functions as even, odd, or neither. ☐ Calculate zeros of functions. ☐ Identify extrema and use them to identify intervals of increasing decreasing. ☐ Classify types of discontinuity. ☐ Describe the end behavior of functions.	**Vocabulary:** symmetry, even/odd functions, zeros, relative and absolute extrema, intervals of increasing and decreasing, discontinuity, end behavior	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
How can operations be used to combine functions algebraically and graphically? **2 Days**	☐ Add, subtract, multiply, divide, and compose functions. ☐ Evaluate, write, and graph piecewise functions.	**Vocabulary:** arithmetic combinations**,** piecewise functions, function composition, difference quotient	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
How do you find the inverse relation of a function and determine if it is a one-to-one function? **1 Day**	☐ Find inverse functions algebraically. ☐ Verify two (2) functions are inverses using composition. ☐ Determine graphically if a function has an inverse.	**Vocabulary:** inverse function, horizontal line test, one-to-one function	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.

UNIT 2

Unit Title	Polynomial and Rational Functions		
Unit Description	Students will learn the key features of polynomial functions, including end behavior, zeros, and factoring. Students will also learn the features of rational functions, including asymptotes and holes, and how those features impact graph behavior. These functions are often used to model real-life phenomena.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How is the behavior of Quadratic Functions used to model real world phenomena? **1 Day**	☐ Write and graph quadratic functions in standard and vertex forms. ☐ Apply minimums and maximums values of quadratic functions to real life situations.	**Vocabulary:** polynomial function, quadratic function, axis of symmetry, standard form, vertex	CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.
How can the zeros in factored form of a Polynomial Function be used to describe the behavior of its graph? **2 Days**	☐ Understand the relationship between the degree and leading coefficient of a polynomial to describe the end behavior. ☐ Graph polynomial functions given the zeros and multiplicity. ☐ Apply the Fundamental Theorem of Algebra to determine the number and type of zeros of a polynomial function. ☐ Write polynomial equations given real or imaginary zeros.	**Vocabulary:** leading coefficient test, end behavior, degree, multiplicity, fundamental theorem of algebra	CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.

How do you use polynomial dividing techniques to find factors of polynomials? **1 Day**	☐ Use long division and synthetic division to change a standard form polynomial into its factored form. ☐ Use the Remainder Theorem and Factor Theorem to determine if a divisor is a factor of a polynomial.	**Vocabulary:** long division, synthetic division, Remainder Theorem, Factor Theorem	CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.
How do you find the zeros of a polynomial function? **2 Days**	☐ Solve for the real and imaginary zeros of polynomial functions.	**Vocabulary:** fundamental theorem of algebra, rational zero test, conjugate, Descartes' rule of signs	CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
How do you graph and describe the features of a Rational Function? **1 Day**	☐ Algebraically and graphically distinguish between holes and vertical asymptotes for rational functions. ☐ Write equations of horizontal and slant asymptotes. ☐ Graph rational functions. ☐ Identify the domain and range of a rational function.	**Vocabulary:** rational function, asymptote, slant asymptote, holes	CC.2.2.HS.D.6 Extend the knowledge of rational functions to rewrite in equivalent forms.
How do you graph an solve nonlinear inequalities? **1 Day**	☐ Graph polynomial and rational inequalities. ☐ Solve polynomial and rational inequalities.	**Vocabulary:** polynomial inequality, rational inequality	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.

How do you decompose a rational expression into a sum of partial fractions? **1 Day**	☐ Decompose rational expressions into a sum of partial fractions.	**Vocabulary:** partial fraction, partial fraction, decomposition, non-repeating linear factors	CC.2.2.HS.D.6 Extend the knowledge of rational functions to rewrite in equivalent forms.

UNIT 3

Unit Title	Matrices		
Unit Description	Students will learn that matrices are used to model and solve a variety of problems. For instance, you use matrices to solve systems of linear equations. Matrices are used to model inventory levels, electrical networks, investment portfolios, and other real-life situations.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How to operations with matrices compare to operations with real numbers? **1 Day**	☐ Perform operations with matrices to include add, subtract, and scalar multiples. ☐ Understand when matrix multiplication can and cannot be done and perform the operation.	**Vocabulary:** scalar, zero matrix, identity matrix	CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
How can systems of equations be setup, solved, and interpreted using matrices? **2 Days**	☐ Solve systems of equations using matrices. ☐ Interpret solutions to systems of equations using a reduced row-echelon form matrix.	**Vocabulary:** matrix, square, diagonal, row, column, reduced row-echelon form	CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
How do we find the determinant and inverse of a square matrix and what are they used for? **2 Days**	☐ Calculate a determinant of a square matrix. ☐ Use the determinant to determine if the inverse of a square matrix exists. ☐ Find the inverse of a square matrix.	**Vocabulary:** inverse matrix, invertible, singular	CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

How do we use matrices and determinants to solve real world problems? **2 Days**	☐ Use matrices to find the area of a triangle given three vertices. ☐ Verify three points are collinear using matrices.	**Vocabulary:** collinear	CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

UNIT 4

Unit Title	Exponential and Logarithmic Functions		
Unit Description	Students will learn that exponential functions involve a constant base and a variable exponent. The inverse of an exponential function is a logarithmic function. Exponential and logarithmic functions are widely used in describing economic and physical phenomena such as compound interest, population growth, memory retention, and decay of radioactive material.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How do exponential functions compare to polynomial functions? **1 Day**	☐ Evaluate and graph an exponential function. ☐ Use exponential growth or decay to model real world scenarios.	**Vocabulary:** exponential function, one-to-one property, natural base, compound interest, radioactive decay	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.

How do logarithmic functions compare to exponential functions? **2 Days**	☐ Evaluate and graph a logarithmic function.	**Vocabulary:** logarithmic function, one-to-one property, natural logarithm	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
How do properties of logarithms compare to properties of exponents? **2 Days**	☐ Use the product, quotient, and power properties to write log expressions in equivalent forms.	**Vocabulary:** change of base, expand, condense	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
How do you solve exponential and logarithmic equations using inverse operations? **1 Day**	☐ Use properties of inverses to solve exponential and logarithmic equations. ☐ Consider the domain of a logarithmic function to determine if solutions when solving are extraneous.	**Vocabulary:** inverse, extraneous solution	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.

How can exponential and logarithmic equations be used to model real world scenarios? **2 Days**	☐ Use exponential growth or decay to model real world scenarios.	**Vocabulary:** exponential model, logarithmic model, radioactive decay, carbon dating, gaussian model, logistic model	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.

UNIT 5

<table>
<tr><td>Unit Title</td><td colspan="3">Introduction to Trigonometry</td></tr>
<tr><td>Unit Description</td><td colspan="3">Students will learn that trigonometry is used to find relationships between the sides and angle of triangles, and to write trigonometric functions as models of real-life quantities. Trigonometric functions are used to model quantities that are periodic. For instance, throughout the day, the depth of water along a shoreline varies with the tides. The depth can be modeled by a trigonometric function.</td></tr>
<tr><td>Unit Assessment</td><td colspan="3">Unit assessment</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td>Essential Question</td><td>Learning Goals</td><td>Content and Vocabulary</td><td>Standards</td></tr>
<tr><td>How can the definitions of trig ratios be used to help find their function values in a right triangle?

2 Days</td><td>☐ Determine the Sine, Cosine, and Tangent values for 30, 45, and 60 degrees using special right triangles.
☐ Use right triangles to evaluate trig functions.</td><td>Vocabulary:
hypotenuse, opposite side, adjacent side, sine, cosine, tangent ratios</td><td>CC.2.2.HS.C.7
Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.

CC.2.3.HS.A.7
Apply trigonometric ratios to solve problems involving right triangles.</td></tr>
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How are radians and degrees related to each other and how do they help us measure angles? **2 Days**	☐ Recognize and use the vocabulary of angles. ☐ Convert between degree and radian measure. ☐ Locate and sketch angles in standard position on a circle. ☐ Find complements, supplements, and coterminal angles. ☐ Calculate arc length and area of a sector. ☐ Convert between linear and angular speed.	**Vocabulary:** initial side, terminal side, standard position, quadrantal angle, coterminal angles, complement, supplement, arc length, linear vs. angular speed, area of a sector	CC.2.2.HS.C.7 Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.
How are trigonometric functions, special right triangles and the unit circle related? **3 Days**	☐ Construct the unit circle using special right triangles to generate the coordinates associated with each angle. ☐ Define trig functions of real numbers using the unit circle. ☐ Use periodic properties to identify patterns in the unit circle.	**Vocabulary:** unit circle, cosecant, secant, cotangent, domain, period	CC.2.2.HS.C.7 Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.
How are the quadrant in which theta lies and the sign of a trigonometric function related? **2 Days**	☐ Find and use reference angles to evaluate trig functions. ☐ Evaluate trig functions of non-unit circle angles.	**Vocabulary:** reference angle	CC.2.2.HS.C.7 Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.

UNIT 6

Unit Title	Trigonometric Graphs		
Unit Description	Students will learn to graph the six trigonometric functions as parent graphs and learn the basic transformations. Then, students will study the graphs of the three inverse trigonometric functions, and learn about real-life applications of trigonometric graphs.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How do you identify the six trigonometric parent graphs and how do they compare? **2 Days**	☐ Graph the 6 trig function parent graphs. ☐ Understand the relationship between the max, min, zeros/asymptotes of a trig function and the unit circle. ☐ Understand the relationship between the max, min, zeros/asymptotes of a trig function to another trig function.	**Vocabulary:** sine unction, cosine function, tangent function, reciprocal functions, cosecant function, secant function, cotangent function	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

How do transformations of trigonometric functions compare to transformations of other algebraic functions? **3 Days**	☐ Transform parent trig functions using the form Asin(Bx-C)+D. ☐ Understand the periodic behavior of trig graphs.	**Vocabulary:** amplitude, period, phase shift, vertical shift	CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions. CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
How are the graphs of the inverse trig functions similar and different to the original trig functions? **2 Days**	☐ Use composition of trig functions to evaluate expressions. ☐ Use inverse trig functions to solve for angle measures. ☐ Graph inverse trig functions over the appropriate domain.	**Vocabulary:** inverse trigonometric function, arcsine function, arccosine function, arctangent function, composition of trig functions	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
How do you apply and model Trigonometric Graphs for real-world application? **2 Days**	☐ Model real world periodic scenarios using sinusoidal functions.	**Vocabulary:** simple harmonic motion, population model	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.

UNIT 7

Unit Title	Trigonometric Identities		
Unit Description	Students will learn that analytic trigonometry is used to simplify trigonometric expressions and solve trigonometric equations. Analytic Trigonometry is used to model real-life phenomena. For instance, when an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. Concepts of trigonometry can be used to describe the apex angle of the cone.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How do you verify fundamental trig identities using fundamental formulas? **2 Days**	☐ Recognize and write the fundamental trigonometric identities. ☐ Use the fundamental trigonometric identities to evaluate, simplify, and rewrite trigonometric functions.	**Vocabulary:** reciprocal identities, quotient identities, Pythagorean identities	CC.2.2.HS.C.9 Prove the Pythagorean identity and use it to calculate trigonometric ratios. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.
How do you verify trig identities? **2 Days**	☐ Verify trigonometric identities. ☐ Use trigonometric identities to rewrite trigonometric equations that model real-life situations.	**Vocabulary:** verify trig identities	CC.2.2.HS.C.9 Prove the Pythagorean identity and use it to calculate trigonometric ratios. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.

How do you use algebraic techniques to solve trig equations? **2 Days**	☐ Use standard algebraic techniques to solve trigonometric equations. ☐ Solve trigonometric equations of quadratic type. ☐ Solve trigonometric equations involving multiple angles. ☐ Use inverse trigonometric functions to solve trigonometric equations.	**Vocabulary:** trigonometric equation	CC.2.2.HS.C.9 Prove the Pythagorean identity and use it to calculate trigonometric ratios. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.
How do you use sum and difference identities to evaluate trigonometric functions, verify identities, and solve trigonometric equations? **2 Days**	☐ Use sum and difference formulas to evaluate trigonometric functions. ☐ Use sum and difference formulas to verify identities. ☐ Use sum and difference formulas to solve trigonometric equations.	**Vocabulary:** sum and difference formulas	CC.2.2.HS.C.9 Prove the Pythagorean identity and use it to calculate trigonometric ratios. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.

How do you use advanced identities (multiple-angle, power-reducing, half-angle, and product-to-sum) to rewrite and evaluate trigonometric functions? **2 Days**	☐ Use multiple-angle formulas to rewrite and evaluate trigonometric functions. ☐ Use power-reducing formulas to rewrite and evaluate trigonometric functions. ☐ Use half-angle formulas to rewrite and evaluate trigonometric functions. ☐ Use product-to-sum formulas to rewrite and evaluate trigonometric functions. ☐ Use trigonometric formulas to rewrite real-life models.	**Vocabulary:** multiple-angle formulas, power-reducing formulas, half-angle formulas, product-to-sum formulas	CC.2.2.HS.C.9 Prove the Pythagorean identity and use it to calculate trigonometric ratios. CC.2.3.HS.A.7 Apply trigonometric ratios to solve problems involving right triangles.

UNIT 8

Unit Title	Polar Graphs		
Unit Description	Students will learn that a polar curve is a shape constructed using the polar coordinate system. Polar curves are defined by points that are a variable distance from the origin (the pole) depending on the angle measured off the positive x-axis. Polar curves can describe familiar Cartesian shapes such as ellipses as well as some unfamiliar shapes such as cardioids.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How are polar coordinates graphed on the coordinate plane? **2 Days**	☐ Plot points on the polar coordinate system.	**Vocabulary:** polar coordinate system, pole/origin, polar axis	FUN-3.G.1 Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates.
How do you convert points between polar and rectangular units? **2 Days**	☐ Convert points from rectangular to polar form and vice versa.	**Vocabulary:** multiple representations of points, polar-to-rectangular conversion, rectangular-to-polar conversion	FUN-3.G.1 Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates.

How do you recognize the graphs of polar equations? **3 Days**	☐ Convert equations from rectangular to polar form and vice versa. ☐ Know the types of graphs associated with different types of polar equations, including circles, cardioids, limaçons, roses, Archimedean spirals, and conic sections.	**Vocabulary:** equation conversion, circle, cardioid, limaçon, rose, Archimedean spiral, conic section	FUN-3.G.1 Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates.

UNIT 9

Unit Title	Sequences, Series, and Probability
Unit Description	Students will learn that sequences and series are used to describe algebraic patterns. Mathematical induction is used to prove formulas. The Binomial Theorem is used to calculate binomial coefficients. Probability theory is used to determine the likelihood of an event. The concepts discussed in this chapter are used to model depreciation, sales, compound interest, population growth, and other real-life applications.
Unit Assessment	Unit assessment

Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
How do you represent the terms of sequences and series in multiple ways? **2 Days**	☐ Represent the terms of a sequence explicitly and recursively. ☐ Know that the Fibonacci Sequence is a common infinite sequence. ☐ Understand how to write infinite and finite series in sigma notation.	**Vocabulary:** infinite and finite sequences, recursive definition of a sequence, Fibonacci Sequence, factorial, summation/sigma notation, infinite and finite series, partial sum	CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
What are the properties of arithmetic sequences and their partial sums? **1 Day**	☐ Understand that in an arithmetic sequence, the difference between any two consecutive terms is always the same number. ☐ Understand that an arithmetic sequence can be defined explicitly by describing its *n*th term with a formula using *n* or recursively by stating its first terms and a formula for its *n*th term.	**Vocabulary:** arithmetic sequence, common difference, *n*th term, sum of a finite arithmetic sequence, partial sum of an arithmetic sequence	CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.

How do you model geometric sequences and series, and find their sums? **2 Days**	☐ Find a specific term of a geometric sequence when they know a term and the common ratio. ☐ Know how to model a geometric sequence, a geometric series, and the sum of a series.	**Vocabulary:** geometric sequence, common ratio, sum of a finite geometric sequence, geometric series, sum of an infinite geometric sequence	CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
How do you use the Binomial Theorem to determine the coefficients of binomial expansions? **1 Day**	☐ Use Pascal's triangle to determine the coefficients of binomial expansions.	**Vocabulary:** binomial coefficients, binomial theorem, Pascal's Triangle, expanding a binomial	CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.
How do you use combinations and permutations to solve counting problems? **1 Day**	☐ Use the fundamental counting principle to solve counting problems. ☐ Use combinations and permutations to solve counting problems.	**Vocabulary:** fundamental counting principle, permutation, combination	CC.2.4.HS.B.6 Use the concepts of independence and conditional probability to interpret data.
How are the concepts of probability used to find the likelihood of different types of events? **1 Day**	☐ Find theoretical and experimental probability of events. ☐ Find probabilities of independent and dependent events. ☐ Find probabilities of the complements of events.	**Vocabulary:** theoretical vs. experimental probability, outcomes, sample space, event, probability, mutually exclusive, independent vs. dependent event, complement of an event	CC.2.4.HS.B.6 Use the concepts of independence and conditional probability to interpret data. CC.2.4.HS.B.7 Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

UNIT 10

Unit Title	Introduction to Calculus		
Unit Description	Students will learn that limits are the foundations of Calculus. Limits help to determine the value of a function or the slope of a function at a certain point when it cannot be solved directly. Real-world objects don't teleport; they move through intermediate positions along their path from A to B. If we miss part of a video because it is buffering, we can use the concept of limits to determine what happened during the missing chunk of time.		
Unit Assessment	Unit assessment		
Essential Question	**Learning Goals**	**Content and Vocabulary**	**Standards**
What are the basic properties of limits? **1 Day**	☐ Use limits to determine a value that a function is approaching. Students will use limits to determine maximum area. ☐ Know the conditions under which a limit does not exist.	**Vocabulary:** limit	CHA-1.A.1 Calculus uses limits to understand and model dynamic change. CHA-1.A.2 Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.

How do you use direct substitution techniques to solve limits? **1 Day**	☐ Use direct substitution methods to solve limits. ☐ Understand the properties of limits. ☐ Solve limits of polynomial and rational functions.	**Vocabulary:** direct substitution, properties of limits	LIM-1.D.1 One-sided limits can be determined analytically or graphically. LIM-1.D.2 Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.
How are common techniques of evaluating limits used to solve limits? **1 Day**	☐ Recognize when a limit statement is in indeterminate form. ☐ Know how to use the "dividing out" and "rationalizing" techniques to solve limits in indeterminate form.	**Vocabulary:** indeterminate form, "dividing out" technique, "rationalizing" technique	LIM-1.E.1 It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.
How can one-sided limits be used to determine the value or existence of any limit? **1 Day**	☐ Know how to evaluate one-sided limits. ☐ Know why one-sided limits are helpful in evaluating the two-sided limits of a function at a point. ☐ Understand the definition of a limit that includes one-sided limits.	**Vocabulary:** one-sided limit, left side limit, right side limit	LIM-1.D.1 One-sided limits can be determined analytically or graphically.

ACCOMMODATIONS AND MODIFICATIONS

Adaptations or modifications to this planned course will allow exceptional students to earn credits toward graduation or develop skills necessary to make a transition from the school environment to community life and employment. The I.E.P. team has determined that modifications to this planned course will meet the student's I.E.P. needs.

<u>Adaptations/Modifications may include but are not limited to:</u>

INSTRUCTION CONTENT

- Modification of instructional content and/or instructional approaches
- Modification or deletion of some of the essential elements

SETTING

- Preferential seating

METHODS

- Additional clarification of content
- Occasional need for one to one instruction
- Minor adjustments or pacing according to the student's rate of mastery
- Written work is difficult, use verbal/oral approaches
- Modifications of assignments/testing
- Reasonable extensions of time for task/project completion
- Assignment sheet/notebook
- Modified/adjusted mastery rates
- Modified/adjusted grading criteria
- Retesting opportunities

MATERIALS

- Supplemental texts and materials
- Large print materials for visually impaired students
- Outlines and/or study sheets
- Manipulative learning materials
- Calculator